

6.1) Величине $A(j, k, l, m)$, $B(j, k, m)$, $C(j, k, m, n)$ се трансформирају по законима:

$$a) \bar{A}(p, q, r, s) = \frac{\partial x^j}{\partial \bar{x}^p} \cdot \frac{\partial \bar{x}^k}{\partial x^l} \cdot \frac{\partial \bar{x}^m}{\partial x^n} \cdot \frac{\partial \bar{x}^s}{\partial x^m} A(j, k, l, m)$$

Одговор је: $\bar{A}_{pqr}^{s} = \frac{\partial x^j}{\partial \bar{x}^p} \cdot \frac{\partial \bar{x}^k}{\partial x^l} \cdot \frac{\partial \bar{x}^m}{\partial x^n} \cdot \frac{\partial \bar{x}^s}{\partial x^m} A_j^{klm} \rightarrow$ тензор

$$b) \bar{B}(p, q, r) = \frac{\partial x^j}{\partial \bar{x}^p} \cdot \frac{\partial \bar{x}^k}{\partial x^l} \cdot \frac{\partial \bar{x}^m}{\partial x^n} B(j, k, m)$$

Одговор је: $\bar{B}_{pq}^r = \frac{\partial x^j}{\partial \bar{x}^p} \cdot \frac{\partial \bar{x}^k}{\partial x^l} \cdot \frac{\partial \bar{x}^m}{\partial x^n} B_{jk}^m \rightarrow$ тензор

$$c) \bar{C}(p, q, r, s) = \frac{\partial \bar{x}^p}{\partial x^j} \cdot \frac{\partial x^k}{\partial \bar{x}^q} \cdot \frac{\partial x^m}{\partial \bar{x}^r} \cdot \frac{\partial x^s}{\partial \bar{x}^n} C(j, k, m, n)$$

$\bar{C}_{pqr}^s = \frac{\partial \bar{x}^p}{\partial x^j} \cdot \frac{\partial x^k}{\partial \bar{x}^q} \cdot \frac{\partial x^m}{\partial \bar{x}^r} \cdot \frac{\partial x^s}{\partial \bar{x}^n} C_{pqr}^s$, иј. треба да је $C(j, q, m, s)$

иа $C(p, q, r, s)$ није тензор!

6.5) Ако је ϕ инваријентна, испитати да ли је $\frac{\partial^2 \phi}{\partial x^i \partial x^j}$ тензор.

$$\begin{aligned}
 \overline{\left(\frac{\partial^2 \phi}{\partial x^i \partial x^j} \right)} &= \frac{\partial^2 \bar{\phi}}{\partial \bar{x}^i \partial \bar{x}^j} = \frac{\partial^2 \phi}{\partial x^i \partial x^j} = \frac{\partial}{\partial x^i} \left(\frac{\partial \phi}{\partial x^j} \right) = \\
 &= \frac{\partial}{\partial x^i} \left(\frac{\partial \phi(x^1, x^2, \dots, x^n)}{\partial x^j} \right) = \frac{\partial}{\partial x^i} \cdot \left(\frac{\partial \phi}{\partial x^k} \cdot \frac{\partial x^k}{\partial x^j} \right) = \\
 &= \frac{\partial}{\partial x^i} \left(\frac{\partial \phi}{\partial x^k} \right) \cdot \frac{\partial x^k}{\partial x^j} + \frac{\partial \phi}{\partial x^k} \cdot \frac{\partial^2 x^k}{\partial x^i \partial x^j} = \\
 &= \frac{\partial}{\partial x^l} \cdot \frac{\partial x^l}{\partial x^i} \left(\frac{\partial \phi}{\partial x^k} \right) \frac{\partial x^k}{\partial x^j} + \frac{\partial \phi}{\partial x^k} \cdot \frac{\partial^2 x^k}{\partial x^i \partial x^j} = \\
 &= \frac{\partial}{\partial x^l} \left(\frac{\partial \phi}{\partial x^k} \right) \frac{\partial x^l}{\partial x^i} \cdot \frac{\partial x^k}{\partial x^j} + \frac{\partial \phi}{\partial x^k} \cdot \frac{\partial^2 x^k}{\partial x^i \partial x^j} = \\
 &= \frac{\partial x^l}{\partial x^i} \cdot \frac{\partial x^k}{\partial x^j} \cdot \frac{\partial^2 \phi}{\partial x^k \partial x^l} + \frac{\partial \phi}{\partial x^k} \cdot \frac{\partial^2 x^k}{\partial x^i \partial x^j} \rightarrow \text{није тензор!}
 \end{aligned}$$